Exercise 59

(a) Prove that the equation has at least one real root. (b) Use your graphing device to find the root correct to three decimal places.

$$100e^{-x/100} = 0.01x^2$$

Solution

Bring both terms to the same side.

$$100e^{-x/100} - 0.01x^2 = 0$$

The function $f(x) = 100e^{-x/100} - 0.01x^2$ is continuous everywhere because it's the difference of two functions known to be continuous everywhere, an exponential function and a polynomial function.

$$f(x) = 0$$

Find a value of x for which the function is negative, and find a value of x for which the function is positive.

$$f(60) \approx 18.9$$

$$f(80) \approx -19.1$$

$f(x)$ is continuous on the closed interval $[60, 80]$, and $N = 0$ lies between $f(60)$ and $f(80)$. By the Intermediate Value Theorem, then, there exists a root within $60 < x < 80$. Find other values of x within this interval for which the function is negative and positive.

$$f(70) \approx 0.659$$

$$f(71) \approx -1.25$$

$f(x)$ is continuous on the closed interval $[70, 71]$, and $N = 0$ lies between $f(70)$ and $f(71)$. By the Intermediate Value Theorem, then, there exists a root within $70 < x < 71$. Find other values of x within this interval for which the function is negative and positive.

$$f(70.346) \approx 0.00141$$

$$f(70.347) \approx -0.000490$$

$f(x)$ is continuous on the closed interval $[70.346, 70.347]$, and $N = 0$ lies between $f(70.346)$ and $f(70.347)$. By the Intermediate Value Theorem, then, there exists a root within $70.346 < x < 70.347$. The function is closer to zero at $x = 70.347$ than it is at $x = 70.346$. Therefore, to three decimal places, the root is

$$x \approx 70.347.$$

This is reflected in the graph of $f(x)$ versus x.